Space Travel Under Constant 1g Acceleration

The basic principle behind every interplanetary space probe ever launched is to accelerate briefly, and then coast, following an elliptical, parabolic, or mildly hyperbolic solar trajectory to your destination, using gravity assists whenever possible. But this is very slow.

Imagine, for a moment, that we have a spacecraft that is capable of a constant 1g ("one gee" = 9.8 m/s^2) acceleration: your spacecraft would accelerate for the first half of the journey, and then decelerate for the second half of the journey for a rendezvous with your destination. A constant 1g acceleration would allow human occupants the comfort of an earthlike gravitational environment where they would not be weightless except during very brief periods during the mission. Granted such a rocket ship would require a tremendous source of power, far beyond what today's chemical rockets can provide, but the day will come, perhaps even in our lifetimes, when probes and people will routinely travel the solar system. Journeys to the stars, however, will be much more difficult.

The key to tomorrow's space propulsion systems will be hydrogen fusion and, later, matter-antimatter annihilation. The fusion of hydrogen into helium provides energy $E = 0.008\ mc^2$. This may not seem like much energy, but when today's technological hurdles are overcome, fusion reactors will produce far more energy in a manner far safer than today's fission reactors. Matter-antimatter annihilation, on the other hand, completely converts mass into energy in the amount given by Einstein's famous equation $E = mc^2$. You cannot get any more energy than this out of any conceivable power, or propulsion, system. Of course, no system is perfect, so there will be some losses that will reduce the efficiency of even the best fusion or matter-antimatter propulsion systems by a few percent.

How long would it take to travel from Earth to the Moon or any of the planets in our solar system under constant 1g acceleration for the first half of the journey and constant 1g deceleration during the second half of the journey? Using the equation below, and the solar system table that follows, you can calculate this easily.

Keep in mind that under a constant 1g acceleration, your velocity quickly becomes so great that you can assume a straight-line trajectory from point **a** to point **b** anywhere in our solar system.

$$T = 2\sqrt{\frac{k}{a}\left[\left(\frac{d}{2k}+1\right)^2 - 1\right]}$$

Idea: use "dimensional analysis" to make sure the calculations on the right side of the equals sign give you the correct units (time in seconds) to match the left side of the equation.

where $k = c^2 / a$
and c = the speed of light = 2.99792458×10^8 m/s *exactly*
and a = the constant acceleration / deceleration
and d = the distance to your destination

Maximum velocity is reached at the halfway point (when you stop accelerating and begin decelerating) and is given by

$$v_{max} = \frac{c}{\sqrt{1 + \dfrac{k}{a\left(\frac{T}{2}\right)^2}}}$$

Again, use "dimensional analysis" to make sure your units come out right.

The energy per unit mass needed for the trip (one way) is then given by

$$E_{kg} = 2c^2\left(\frac{1}{\sqrt{1-\left(\frac{v_{max}}{c}\right)^2}} - 1\right)$$

where E_{kg} is the energy needed per kg of payload (J/kg) to make the journey

How much fuel will you need for the journey?

hydrogen fusion into helium gives: $\mathbf{E_{fusion} = 0.008\, m_{fuel}c^2}$

matter-antimatter annihilation gives: $\mathbf{E_{anti} = m_{fuel}c^2}$

This assumes 100% of the fuel goes into propelling the spacecraft, but of course there will be energy losses which will require a greater amount of fuel than this.

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Mean distance from Earth to the Moon: 384,399 km

Mean distance of the planets from the Sun (in AU)

Mercury	0.39
Venus	0.72
Earth	1.00
Mars	1.52
Jupiter	5.20
Saturn	9.53
Uranus	19.24
Neptune	30.21
Pluto	39.79
Eris	67.67

Distance to the nearest star outside our solar system

Proxima Centauri 4.22 ly

Distance to the nearest large galaxy beyond our Milky Way

M31, the Andromeda Galaxy 2.54 Mly

1 AU = 1.496×10^{11} m

1 ly = 9.461×10^{15} m