Distance and Bearing

To find the shortest distance between any two points on the surface of the Earth (or any other nearly-spherical object) use the following equation:

$$d = r_p \tan^{-1}\left(\frac{\sqrt{\cos^2\varphi_2 \sin^2(\lambda_2-\lambda_1) + [\cos\varphi_1 \sin\varphi_2 - \sin\varphi_1 \cos\varphi_2 \cos(\lambda_2-\lambda_1)]^2}}{\sin\varphi_1 \sin\varphi_2 + \cos\varphi_1 \cos\varphi_2 \cos(\lambda_2-\lambda_1)}\right)$$

where
- d = the line-of-sight distance between any two points (in units of r_p)
- r_p = the radius of the planet (km or miles, or any distance unit) (the mean radius of the Earth is **6371 km**)
- $\tan^{-1}$ = the arctangent function; if using a calculator, *be sure* it is in radians mode when you have it evaluate the arctangent function; if the arctangent is negative, add πr_p to d
- φ_1 = latitude of 1st point in radians (negative south of the equator)
- φ_2 = latitude of 2nd point in radians (negative south of the equator)
- λ_1 = longitude of 1st point in radians (negative west of Greenwich)
- λ_2 = longitude of 2nd point in radians (negative west of Greenwich)

A **radian** is a unit of angular measurement, and is equal to 57.3°. There are 2π radians in a circle, so $2\pi = 360°$.

In addition to knowing the distance from one point to another on the surface of a sphere, you might also be interested in knowing the compass direction, or *bearing*, you must take to get from the first point to the second point.

$$\theta = \frac{180°}{\pi}\tan^{-1}\left(\frac{\cos\varphi_2 \sin(\lambda_2-\lambda_1)}{\cos\varphi_1 \sin\varphi_2 - \sin\varphi_1 \cos\varphi_2 \cos(\lambda_2-\lambda_1)}\right)$$

Greek to Me

θ = theta	φ = phi
π = pi	λ = lambda

where θ = the azimuth angle (east of north) in degrees from the 1st point to the 2nd point (a table relating azimuth to compass bearing is given on the next page); note: if θ is negative, add 360°

Now for an example. Let's calculate the distance between the Kitt Peak National Observatory 4-meter telescope, and the 8.4-meter (x2) Large Binocular Telescope on Mt. Graham. First, the coordinates of these two locations:

KPNO 4m	φ_1 =	31° 57′ 50″ N =	31.9639° =	0.557875 radians
	λ_1 =	111° 36′ 00″ W =	-111.6000° =	-1.947787 radians
LBTO 8.4m	φ_2 =	32° 42′ 05″ N =	32.7014° =	0.570746 radians
	λ_2 =	109° 53′ 36″ W =	-109.8933° =	-1.918000 radians

Plugging these numbers into the distance equation, we get

$$\mathbf{d} = \mathbf{6371\ km}\ \tan^{-1}\frac{\sqrt{\cos^2(0.570746)\sin^2(-1.918+1.947787)+[\cos(0.557875)\sin(0.570746)-\sin(0.557875)\cos(0.570746)\cos(-1.918+1.947787)]^2}}{\sin(0.557875)\sin(0.570746)+\cos(0.557875)\cos(0.570746)\cos(-1.918+1.947787)}$$

And chugging away, we get

d = 180 km = 112 miles

Then, to figure out the line-of-sight direction from Kitt Peak to Mt. Graham, we again substitute the values

$$\theta = \frac{180°}{3.141592654}\tan^{-1}\left(\frac{\cos(0.570746)\sin(-1.918+1.947787)}{\cos(0.557875)\sin(0.570746)-\sin(0.557875)\cos(0.570746)\cos(-1.918+1.947787)}\right)$$

And, again chugging away, we get

$\theta = 62.5°$

which we can see from our table is in an east-northeasterly (ENE) direction.

Azimuth Bearing Table

349 - 11	N
12 - 33	NNE
34 - 56	NE
57 - 78	ENE
79 - 101	E
102 - 123	ESE
124 - 146	SE
147 - 168	SSE
169 - 191	S
192 - 213	SSW
214 - 236	SW
237 - 258	WSW
259 - 281	W
282 - 303	WNW
304 - 326	NW
327 - 348	NNW

By the way, the distance corresponding to 1° of longitude (λ) depends on your latitude (φ) as follows:

$d_\lambda = 111.4133 \cos\varphi - 0.0935 \cos 3\varphi + 0.0001 \cos 5\varphi$ **km**

And, perhaps surprisingly, the distance corresponding to 1° of latitude (φ) also depends (in a small way) on your latitude! This is due to the fact that the Earth is an oblate spheroid and not a perfect sphere. The equation is

$d_\varphi = 111.1334 - 0.5594 \cos 2\varphi + 0.0012 \cos 4\varphi$ **km**

We will leave it as an exercise for the reader to calculate the distance (in miles or kilometers) corresponding to 1°, 1′, and 1″ of latitude and longitude at his or her location.

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