Angular Separation

The angular separation between any two points on the celestial sphere is given by the following equation:

$$
\theta = \left( \frac{180^\circ}{\pi} \right) \tan^{-1} \left( \frac{\sqrt{\cos^2 \delta_2 \sin^2 (\alpha_2 - \alpha_1) + \cos \delta_1 \sin \delta_2 - \sin \delta_1 \cos \delta_2 \cos (\alpha_2 - \alpha_1)}}{\sin \delta_1 \sin \delta_2 + \cos \delta_1 \cos \delta_2 \cos (\alpha_2 - \alpha_1)} \right)
$$

where
- $\theta$ = the angular separation between two points in decimal degrees
- $\tan^{-1}$ = the arctangent function; if using a calculator, *be sure* it is in radians mode when you have it evaluate the arctangent function; if the arctangent is negative, add 180° to $\theta$
- $\alpha_1$ = the right ascension of the 1st point in radians
- $\alpha_2$ = the right ascension of the 2nd point in radians
- $\delta_1$ = the declination of the 1st point in radians
- $\delta_2$ = the declination of the 2nd point in radians

A radian is a unit of angular measurement, and is equal to 57.3°. There are $2\pi$ radians in a circle, so $2\pi = 360^\circ$.

You’ll notice that the equation we use to find the angular separation between two points on the celestial sphere is almost identical to the equation we use to find the shortest distance between two points on the surface of the Earth (see *Distance and Bearing*). The planet radius term $r_p$ becomes unity (i.e. 1) for the celestial sphere, so distances become angles and we can eliminate that term from the equation. Also, we substitute right ascension ($\alpha$) for its terrestrial analogue longitude ($\lambda$), and likewise declination ($\delta$) for its analogue latitude ($\varphi$). That’s all there is to it!

Let’s work an example. It is commonly stated that the angular distance between the pointer stars of the Big Dipper, Merak and Dubhe (pronounced ME-rack and DUB-ee), is 5°. Let’s use our equation to see how accurate that statement is.

**Merak**
- $\alpha_1 = 11^h 01^m 50^s = 11^h.0306 = 165.458^\circ = 2.88779$ radians
- $\delta_1 = +56^\circ 22' 57'' = 56.3825^\circ = 0.984060$ radians

**Dubhe**
- $\alpha_2 = 11^h 03^m 44^s = 11^h.0622 = 165.933^\circ = 2.89608$ radians
- $\delta_2 = +61^\circ 45' 04'' = 61.7511^\circ = 1.07776$ radians

Calculating it out, we get $\theta = 5.37413^\circ = 5^\circ 22' 27''$. Not bad!

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